

# Can Gaia conclude the local dark matter density problem?

---

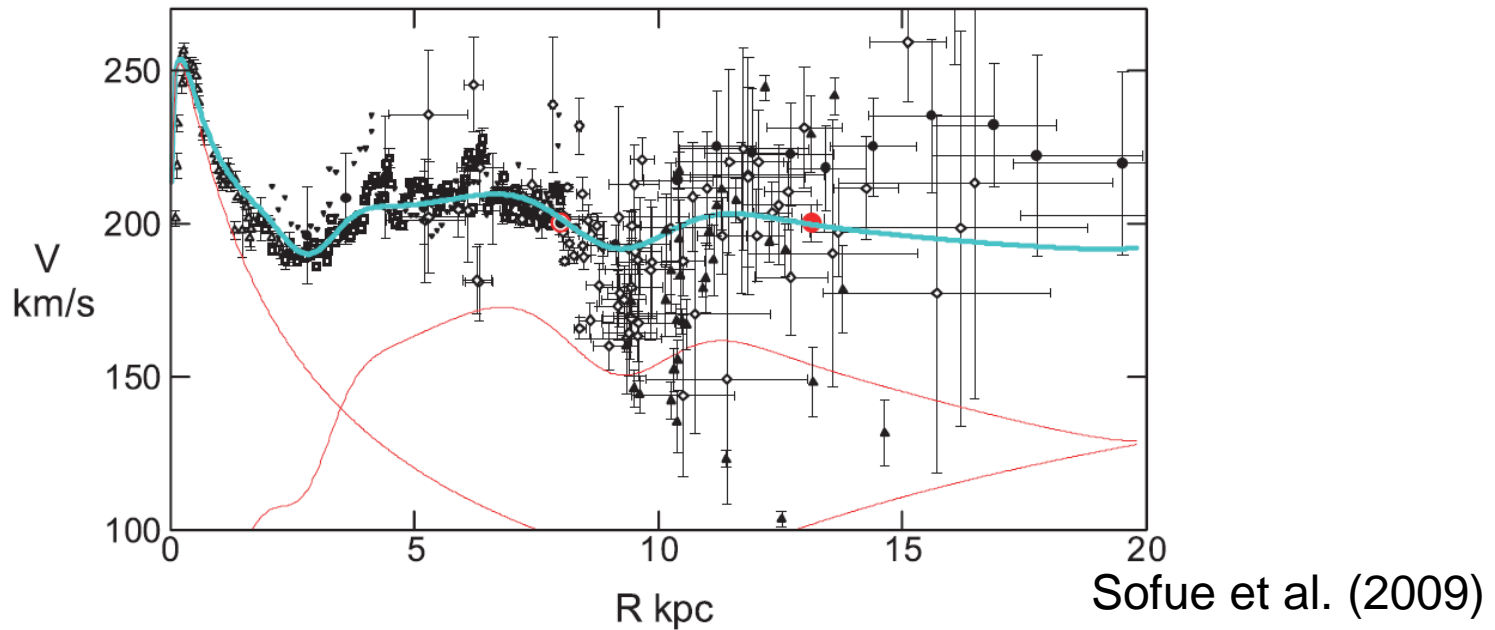
Korea Astronomy and Space Science Institute (KASI)

Shigeki Inoue

**“Astrometric mock observations for determining the local dark matter density”**

**Inoue & Gouda (2013) A&A, 555, A105**

# The Galactic DM halo



- The Galactic rotation curve
  - The analysis has to assume spherical symmetry.
    - The DM density at the solar radius
      - e.g. Sofue (2012)  $\rho_{DM, RC} \cong 0.006 M_{\odot}/pc^3$
  - However, the Galactic halo may not be spherical.

# LDMD determination by vertical motions

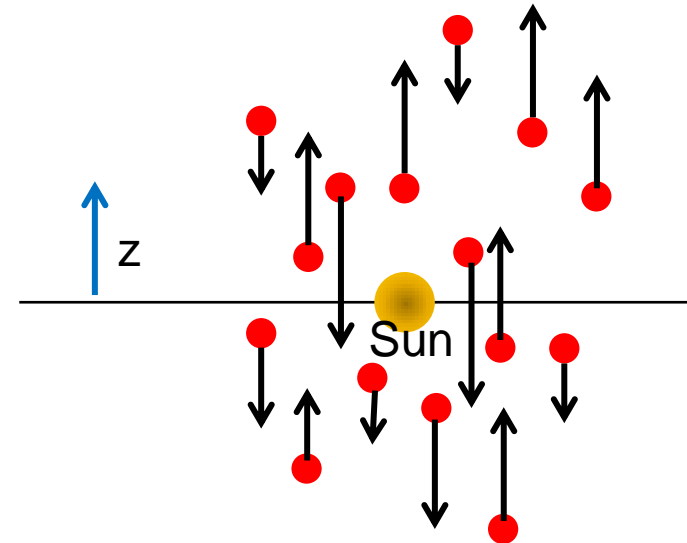
- How is the LDMD determined?

- an old problem; since Oort (1932, 1960), Hill (1960)

- z-Jeans eq. for dynamical tracers

- $\frac{\partial}{\partial z} (\rho \sigma_z^2) + \rho \frac{\partial \Phi}{\partial z} = 0$

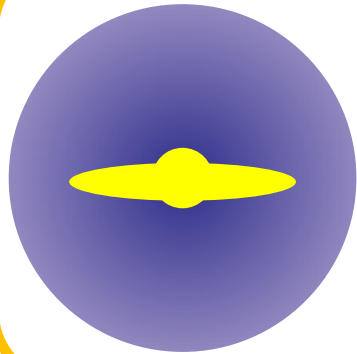
- or Boltzmann eq.



- If we know (or assume)  $\rho$  and  $\sigma_z$ ,  $\Phi$  can be derived.

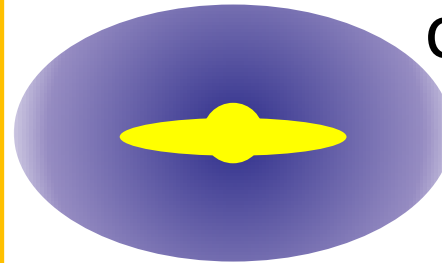
- Baryonic density can be subtracted, then we can determine the DM density.

# DM halo shape and the LDMD



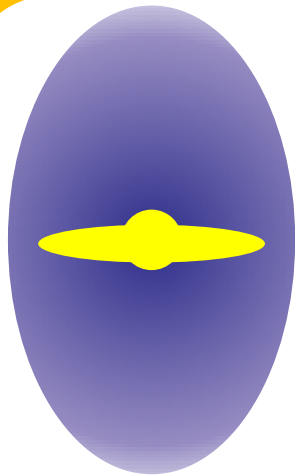
spherical

$$\text{LDMD} = \rho_{DM, RC}$$



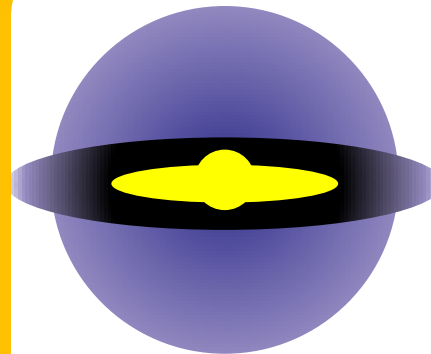
oblate

$$\text{LDMD} > \rho_{DM, RC}$$



prolate

$$\text{LDMD} < \rho_{DM, RC}$$



dark disc

(Read et al. 2009)

$$\text{LDMD} > \rho_{DM, RC}$$

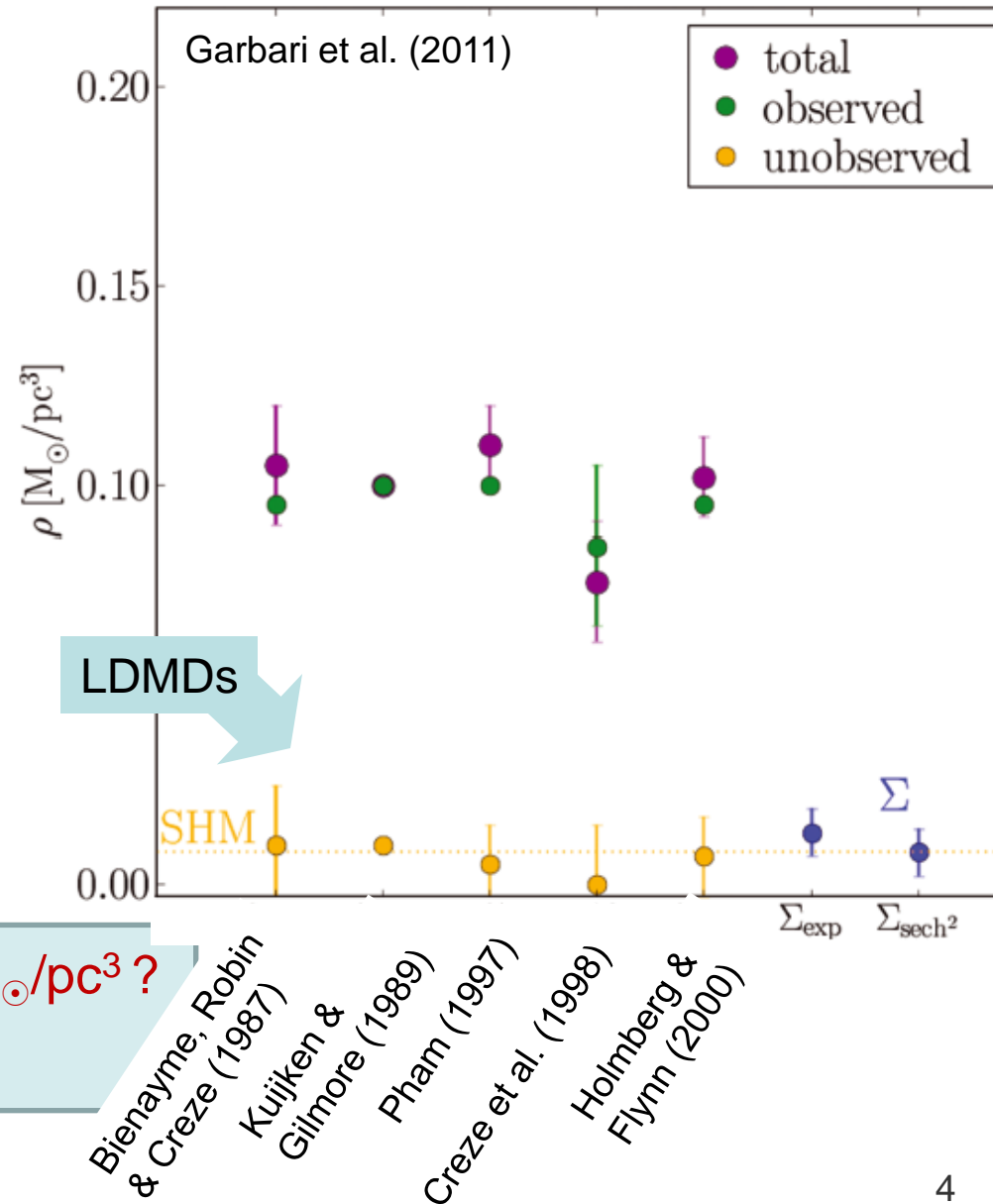
It may give us a hint on the DM halo shape to compare the  $\rho_{DM, RC}$  with the LDMD.

# LDMDs in previous studies

- Garbari et al. (2011)
  - LDMD  $\cong 0.033 M_{\odot}/\text{pc}^3$
- Garbari et al. (2012)
  - LDMD  $\cong 0.022 M_{\odot}/\text{pc}^3$
- Bovy & Tremaine (2012)
  - LDMD  $\cong 0.008 M_{\odot}/\text{pc}^3$
- Smith et al. (2012)
  - LDMD  $\cong 0.015 M_{\odot}/\text{pc}^3$
- Zhan et al. (2013)
  - LDMD  $\cong 0.006 M_{\odot}/\text{pc}^3$

• conversing on  $\sim 0.006 - 0.033 M_{\odot}/\text{pc}^3$ ?

• higher than  $\rho_{DM, RC}$ ? or similar?



# The aim of this study

- This study performs “**mock**” observations of astrometry.
  - to scrutinize a method
    - Minimal Assumption method (Garbari et al. 2011, 2012)
    - Are there intrinsic systematic errors in the method?
  - to estimate observational precisions required to determine the LDMD with accuracy.
    - Were the Hipparcos observations precise enough?
    - Are the Gaia observations precise enough?

- the Minimal Assumption method
  - *cf. Garbari et al. (2011, 2012)*

# The MA method by Gairbari et al. (2011, 2012)

- **The Minimal Assumption (MA) method**

- Assumptions

1. The system is in equilibrium
2. DM density is constant in the region we consider ( $z < 1.2$  kpc)
3. The “tilt” term is negligible in Jeans equation

- ◆ *Step1* Choose trial parameters

- the LDMD;  $\rho_{DM}$
- 15 baryon components;  $\rho_{i,0}, \sigma_{z,i}$  at  $z = 0$

- ◆ *Step2* Solve the equations below, compute the trial potential  $\Phi(z)$

- $\frac{\partial^2 \Phi}{\partial z^2} = 4\pi G(\rho_b + \rho_{DM})$

- $\rho_b = \sum \rho_{i,0} \exp\left(-\frac{\Phi(z)}{\sigma_{z,i}^2}\right)$  ← isothermal disc model



# The MA method (galaxy model)

Flynn et al. (2006)

#	Component	$\rho_i^{\text{ass}}(0)$ [ $M_{\odot} \text{pc}^{-3}$ ] (observation)	$\sigma_{z,i}(0)$ [ $\text{km s}^{-1}$ ] (observation)
1	H <sub>2</sub>	0.021	$4.0 \pm 1.0$
2	HI(1)	0.016	$7.0 \pm 1.0$
3	HI(2)	0.012	$9.0 \pm 1.0$
4	Warm gas	0.0009	$40.0 \pm 2.0$
5	Giants	0.0006	$20.0 \pm 2.0$
6	$M_v < 2.5$	0.0031	$7.5 \pm 2.0$
7	$2.5 < M_v < 3.0$	0.0015	$10.5 \pm 2.0$
8	$3.0 < M_v < 4.0$	0.0020	$14.0 \pm 2.0$
9	$4.0 < M_v < 5.0$	0.0022	$18.0 \pm 2.0$
10	$5.0 < M_v < 8.0$	0.007	$18.5 \pm 2.0$
11	$M_v > 8.0$	0.0135	$18.5 \pm 2.0$
12	White dwarfs	0.006	$20.0 \pm 5.0$
13	Brown dwarfs	0.002	$20.0 \pm 5.0$
14	Thick disk	0.0035	$37.0 \pm 5.0$
15	Stellar halo	0.0001	$100.0 \pm 10.0$

- Densities and velocity dispersions at  $z=0$

# The MA method by Garbari et al. (2011, 2012)

- Necessary observations of tracer stars

- density profile:  $\rho_{trac}(z)$
- velocity dispersion profile:  $\sigma_{z,trac}^2(z)$

- ◆ **Step3** predict the tracer density profile

- input  $\sigma_{z,trac}^2(z)$  and the trial potential  $\Phi(z)$  into Jeans equation,

- $$\rho_{pred}(z) = \rho_{pred}(z_{min}) \frac{\sigma_{z,trac}^2(z_{min})}{\sigma_{z,trac}^2(z)} \exp \left\{ - \int_{z_{min}}^z \frac{1}{\sigma_{z,trac}^2(z')} dz' \right\}$$

↑ non-isothermal

- ◆ **Step4** compare the predicted and the observed density fall-off

- evaluate goodness-of-fit
- MCMC: go back to step1

- Mock galaxy model  
Mock tracer model

# Mock galaxy and tracer models

- I assume “**mock**” density distributions for a galaxy model.
- Density profiles for the 15 baryon components and DM

– baryon  $\rho_i(z) = \rho_{0,i} \operatorname{sech}^2(z/h_{z,i})$

- for each of the 15 components  
with each scale height

– dark matter  $\rho_{DM} = 0.01 M_{\odot}/\text{pc}^3$

#	Component	$\rho_i^{\text{ass}}(0) [M_{\odot} \text{pc}^{-3}]$ (observation)
1	H <sub>2</sub>	0.021
2	HI(1)	0.016
3	HI(2)	0.012
4	Warm gas	0.0009
5	Giants	0.0006
6	$M_v < 2.5$	0.0031
7	$2.5 < M_v < 3.0$	0.0015
8	$3.0 < M_v < 4.0$	0.0020
9	$4.0 < M_v < 5.0$	0.0022
10	$5.0 < M_v < 8.0$	0.007
11	$M_v > 8.0$	0.0135
12	White dwarfs	0.006
13	Brown dwarfs	0.002
14	Thick disk	0.0035
15	Stellar halo	0.0001

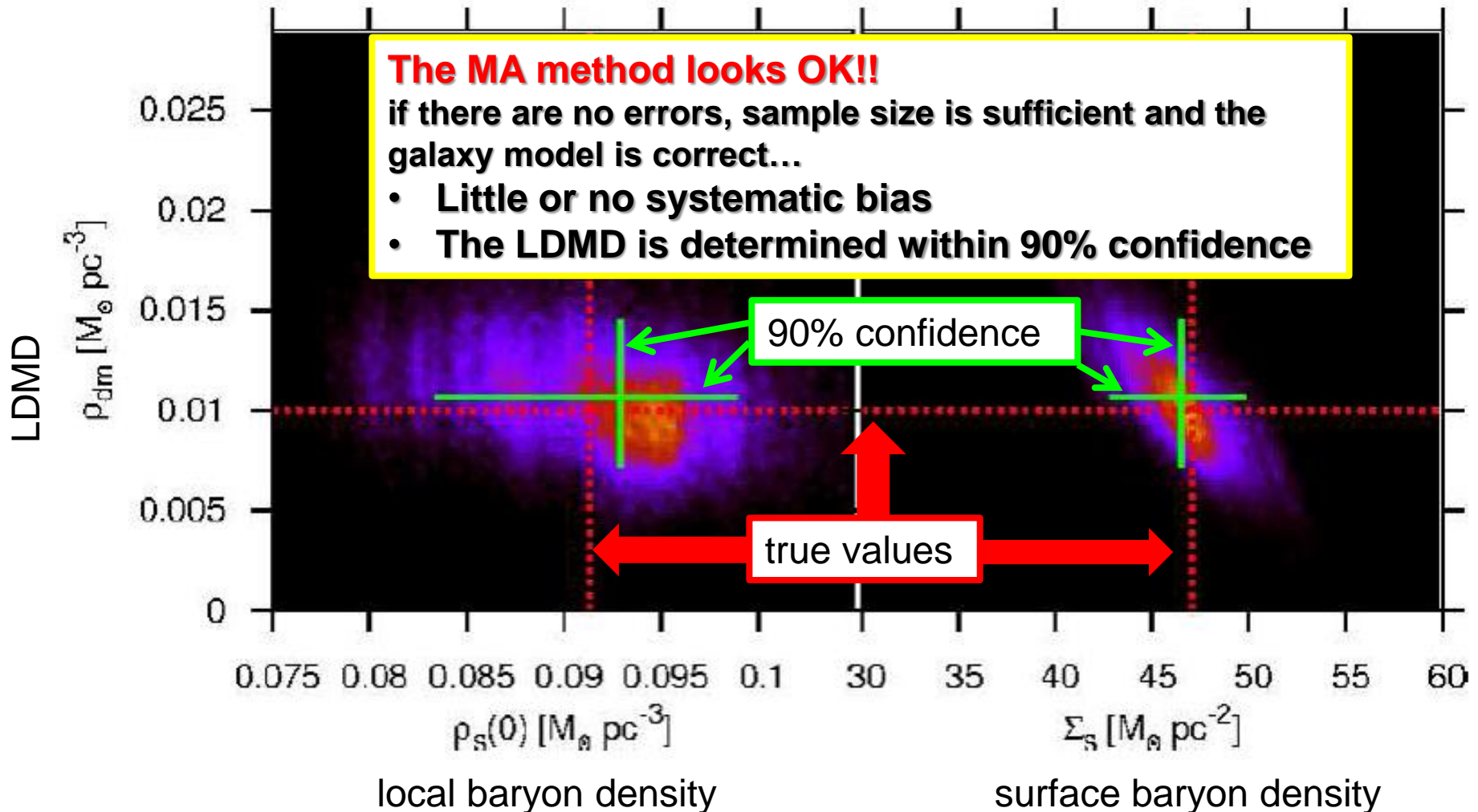
- If a tracer density profile is assumed,  $\sigma_{z,\text{trac}}^2(z)$  can be calculated.

–  $\rho_{\text{trac}}(z) \propto \operatorname{sech}^2(z/400 \text{ pc})$

–  $\sigma_{z,\text{trac}}^2(z) = \frac{1}{\rho_{\text{trac}}} \int_z^{\infty} \rho_{\text{trac}} \frac{d\Phi}{dz'} dz'$

# test of the MA method with the analytic solutions

- MCMC outputs: PDFs of the parameters.



# Mock tracer model & observations

- tracer density distribution
  - $\rho_{trac}(z) \propto \text{sech}^2(z/400 \text{ pc})$ 
    - $z_{min} = 200 \text{ pc}$ ,  $z_{max} = 1.2 \text{ kpc}$

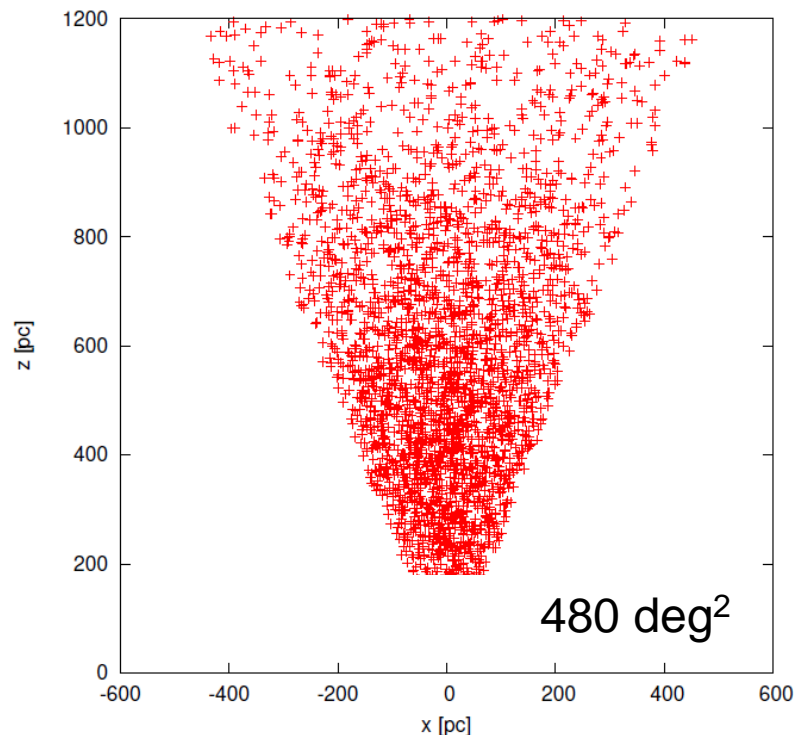
- velocity distributions (Gaussian)

- $\sigma_{z,trac}^2(z) = \frac{1}{\rho_{trac}} \int_z^\infty \rho_{trac} \frac{d\Phi}{dz'} dz'$

- $\sigma_{R,trac}(z) = 40 + 5 \left| \frac{z}{kpc} \right|^{1.5} \text{ [km/s]}$

- $\overline{v_{\phi,trac}}(z) = -19.2 \left| \frac{z}{kpc} \right|^{1.25} \text{ [km/s]}$

- $\sigma_{\phi,trac}(z) = 30 + 3 \left| \frac{z}{kpc} \right|^2 \text{ [km/s]}$



from observations of  
Bond et al. (2010)

The mock observations are 3D, but the variations of  $\rho_{trac}$  and  $\sigma_{trac}$  are 1D.

# Mock tracer model & observations

- tracer density distribution
  - $\rho_{trac}(z) \propto \text{sech}^2(z/400 \text{ pc})$ 
    - $z_{min} = 200 \text{ pc}$ ,  $z_{max} = 1.2 \text{ kpc}$

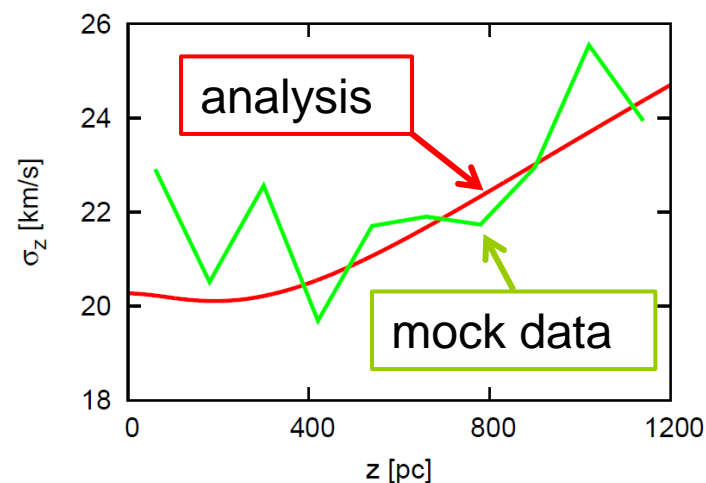
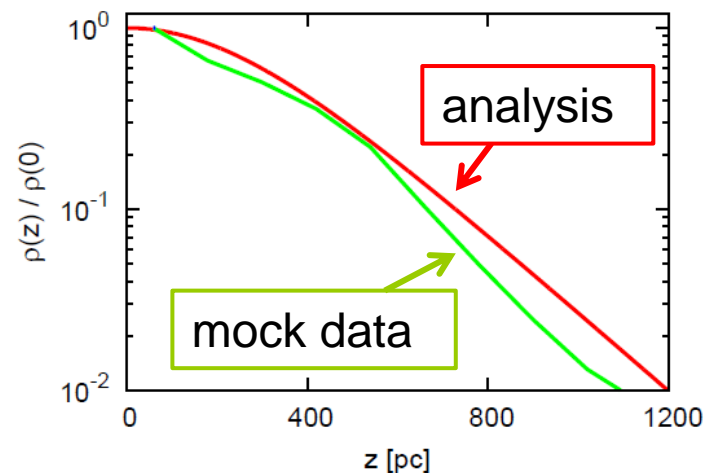
- velocity distributions (Gaussian)

- $\sigma_{z,trac}^2(z) = \frac{1}{\rho_{trac}} \int_z^\infty \rho_{trac} \frac{d\Phi}{dz'} dz'$

- $\sigma_{R,trac}(z) = 40 + 5 \left| \frac{z}{\text{kpc}} \right|^{1.5} \text{ [km/s]}$

- $\overline{v_{\phi,trac}}(z) = -19.2 \left| \frac{z}{\text{kpc}} \right|^{1.25} \text{ [km/s]}$

- $\sigma_{\phi,trac}(z) = 30 + 3 \left| \frac{z}{\text{kpc}} \right|^2 \text{ [km/s]}$

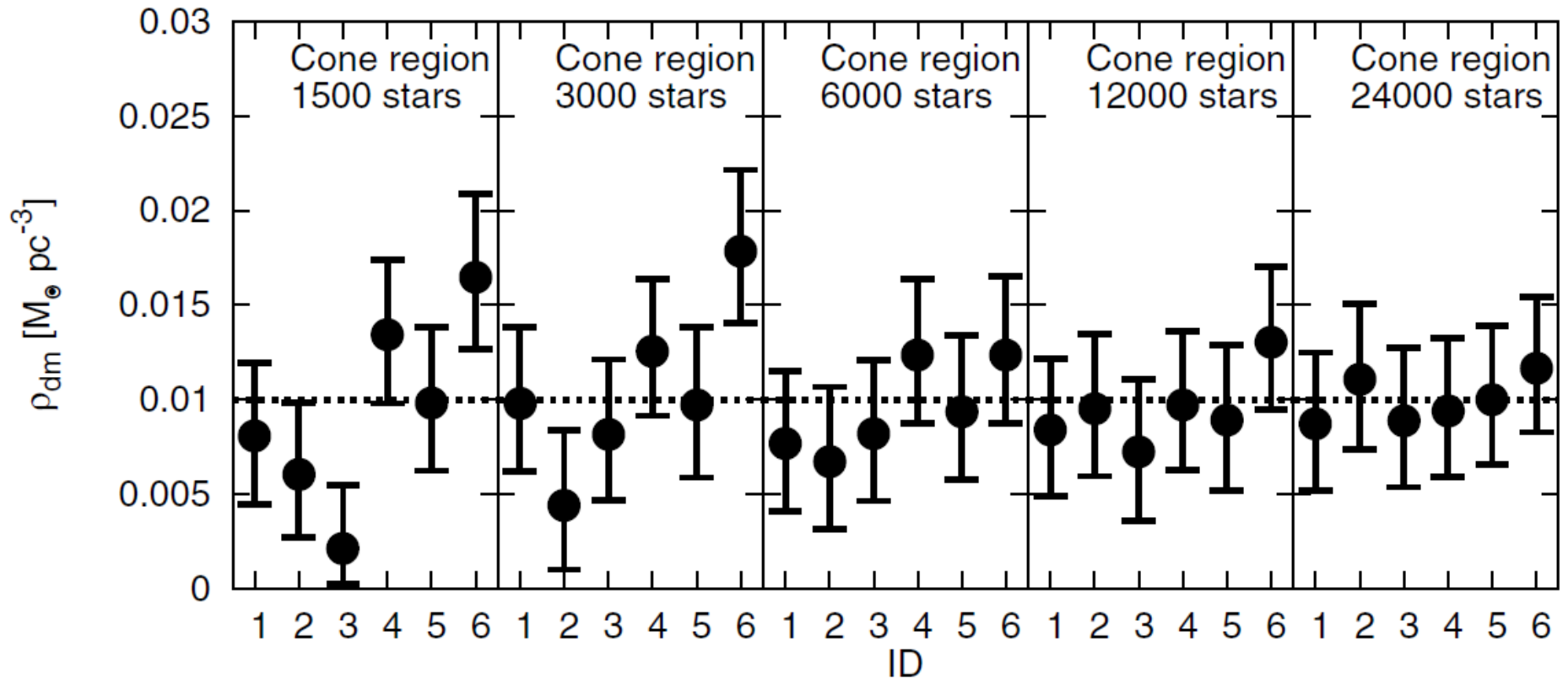


- Mock observations
  - required sample size



# Required sample sizes

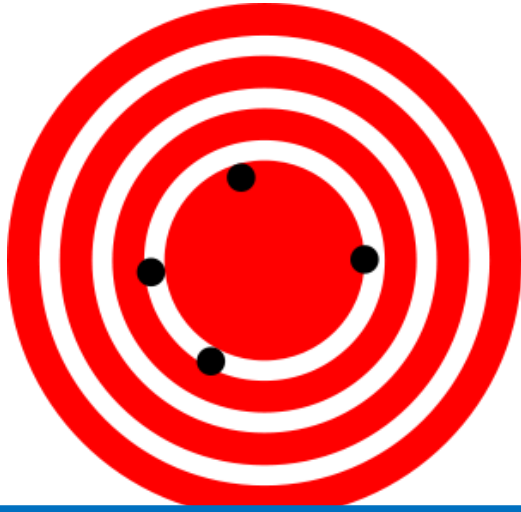
- $\sim > 6000$  stars are required to determine the LDMD



The error bars are indicating 90% confidence levels.

- Mock observations
  - required observational precisions

# What is advantage of astrometry?



imprecise but accurate



inaccurate but precise

- Astrometry is imprecise, but accurate.
  - systematic errors  $\ll$  random errors
  - The errors are already-know.
- In this study, observational uncertainties are assumed to be random Gaussian errors.

# Mock distance errors

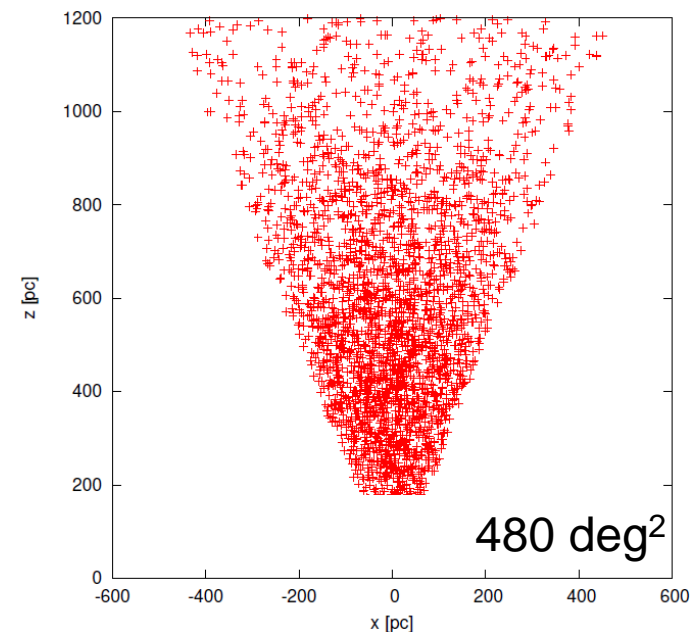
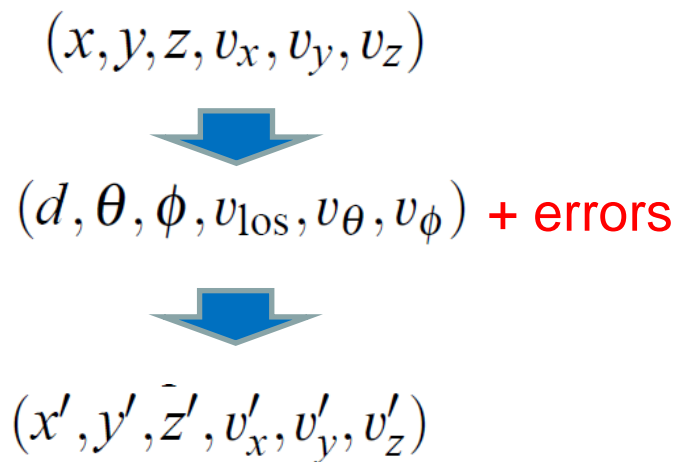
- In astrometry, distance errors can be formalized as follows:
  - Stellar image centroids are determined by photon statistics
    - $\sigma_{\varpi} \propto 1/\sqrt{f} \propto d$
  - fractional parallax error
    - $\sigma_{\varpi}/\varpi \propto d^2$
  - fractional parallax error is equal to fractional distance error (FDE).

$$\varepsilon_{\text{FDE}} = A \left( \frac{d}{\text{kpc}} \right)^2$$

- $A$  corresponds to parallax precision in mas at  $d = 1$  kpc.

# Mock distance errors

- Applying the distance errors to the mock data with Gaussian.

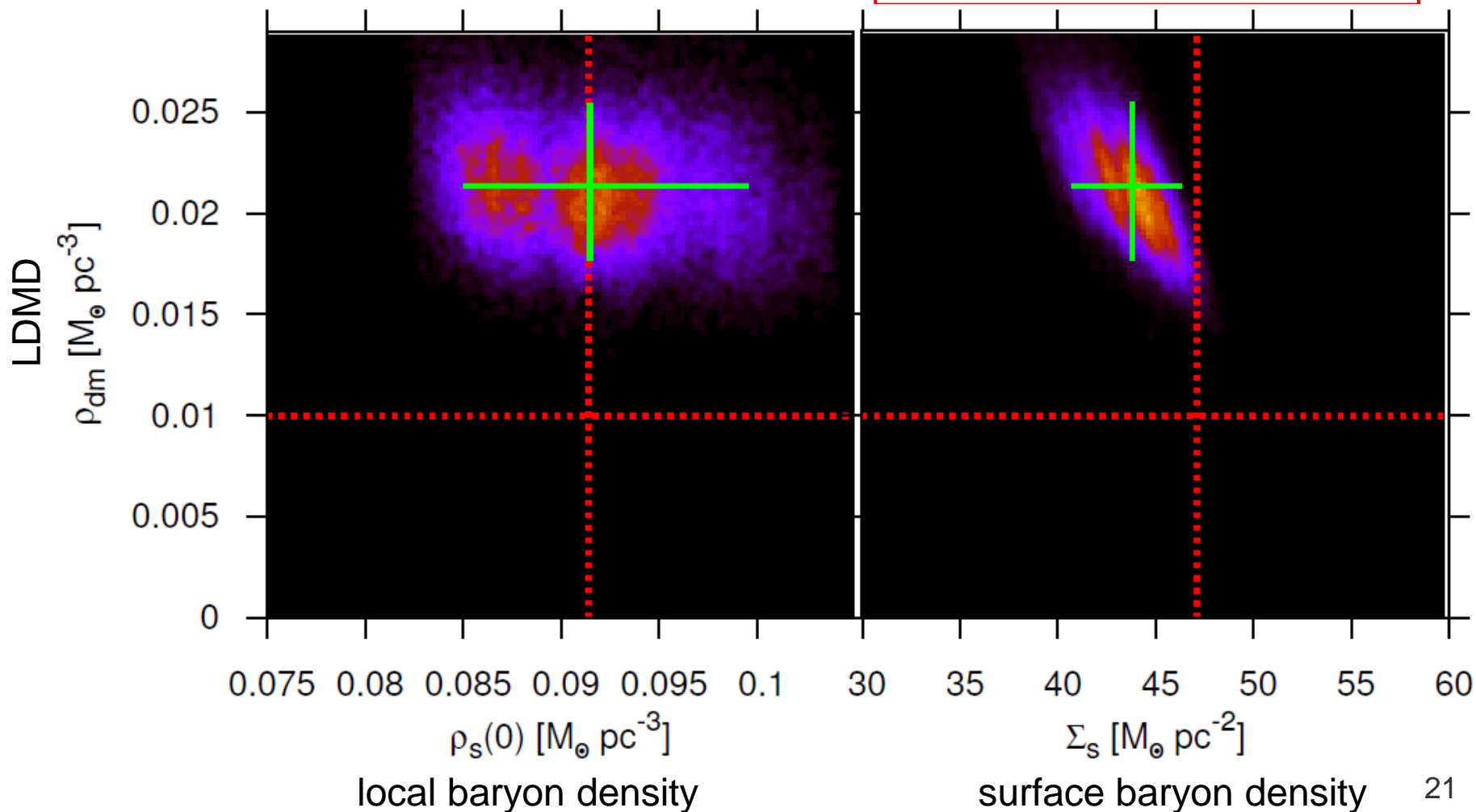


- Moreover, the distance errors propagate to transverse velocities.

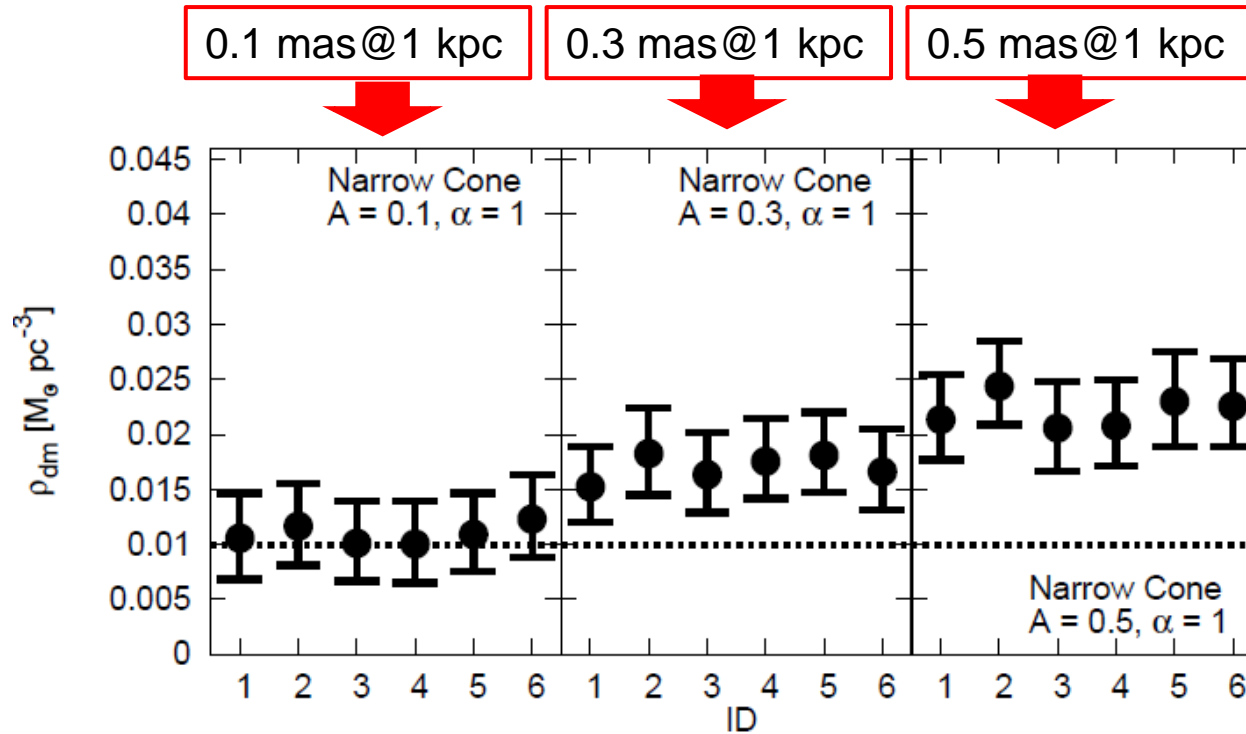
$$v_{\theta} = d \times \mu_{\theta} \qquad v'_{\theta} = v_{\theta} \frac{d'}{d}$$

# Mock distance errors

0.5 mas @ d=1 kpc  
24,000 stars



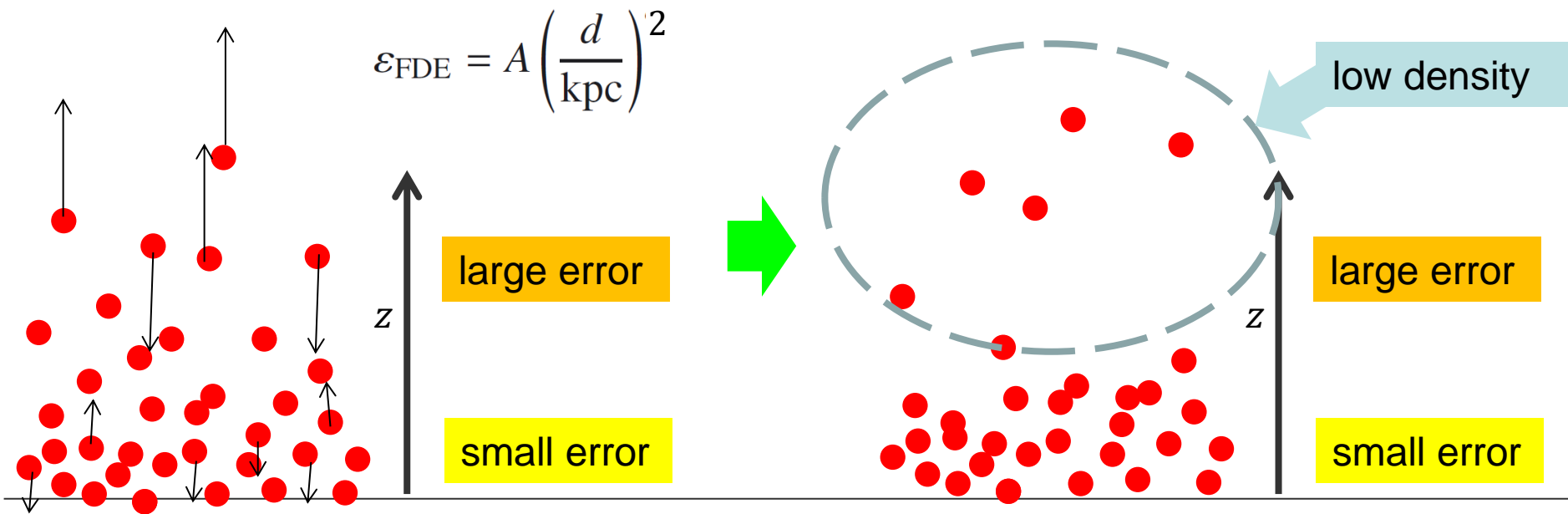
# Mock distance errors



- The parallax errors can cause a systematic overestimate of the LDMD.
- Required parallax precision is **0.1- 0.3 mas @  $d=1$  kpc**
  - *Hipparcos*     $\sim 1\text{mas @ } d=100 \text{ pc}$
  - *Gaia*         $\sim 0.06 \text{ mas @ } d=1\text{kpc for K stars}$

# Why overestimate?

- The distance errors are *random* Gaussian.
  - However, the LDMD was systematically overestimated.



- Large distance errors in high- $z$  region makes the disc thinner than the real.
- The thinner disc prefers a deeper potential.
- The deeper potential means a higher LDMD.



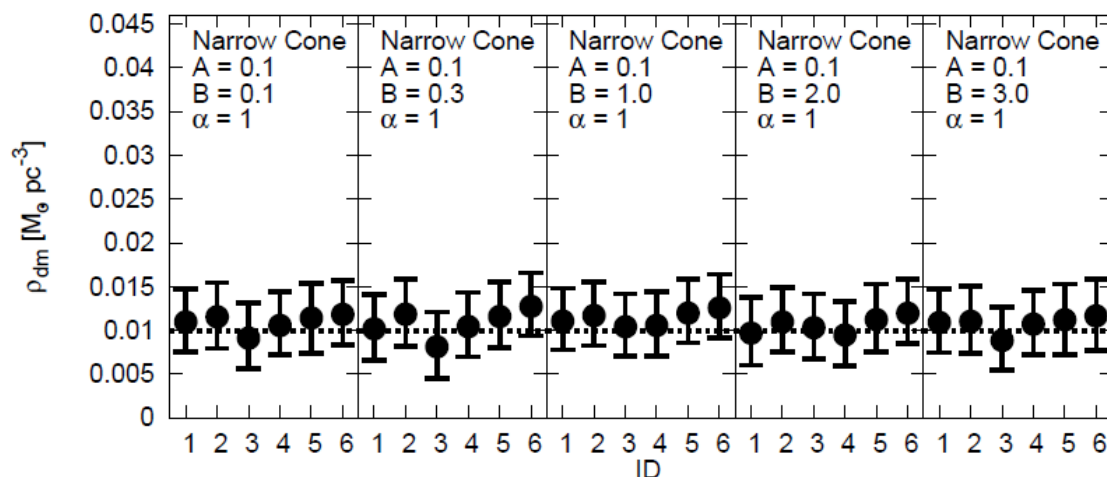
# Proper motion errors

- Proper motion errors also vary with distance in the same way as parallaxes.

$$\varepsilon_{\mu} = B \left( \frac{d}{\text{kpc}} \right) \text{ mas yr}^{-1}$$

3 mas/yr @ 1 kpc

- The proper motion errors little affect the LDMD determinations.
- It is because proper motions have little information about  $v_z$



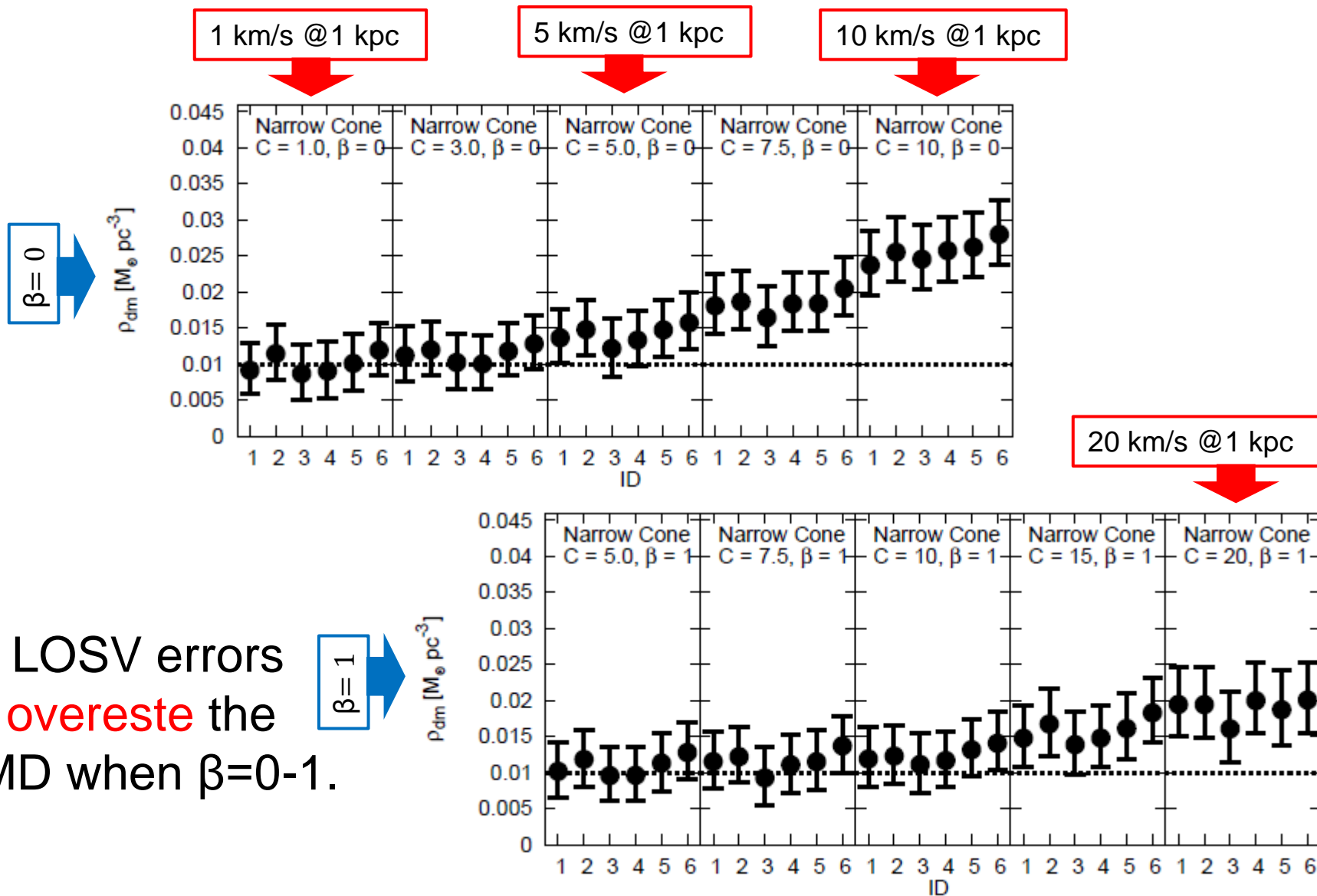
# Mock line-of-sight velocity errors

- LOSV measurements are independent from astrometric observations.
  - *Gaia* is designed to measure LOSVs simultaneously.
- Distance-dependence of LOSV errors is highly complicated.

$$\epsilon_{losv} = C \left( \frac{d}{\text{kpc}} \right)^{\beta} \text{ km s}^{-1} \quad \beta = 0, 1, 2 \text{ or } 3$$

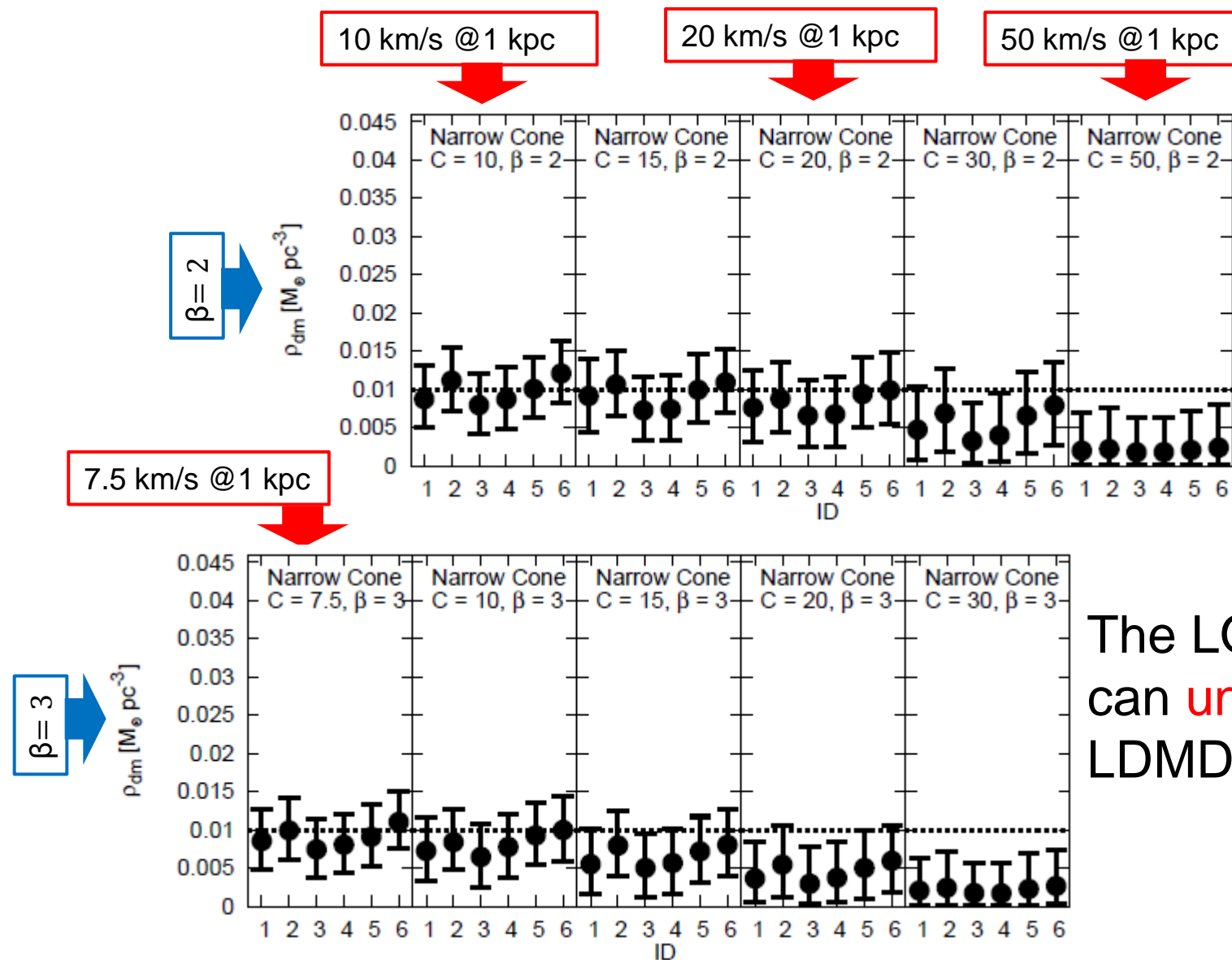
- $\beta = 2 - 3$  is the most likely for *Gaia*.

# Mock line-of-sight velocity errors



The LOSV errors can **overestimate** the LDMD when  $\beta=0-1$ .

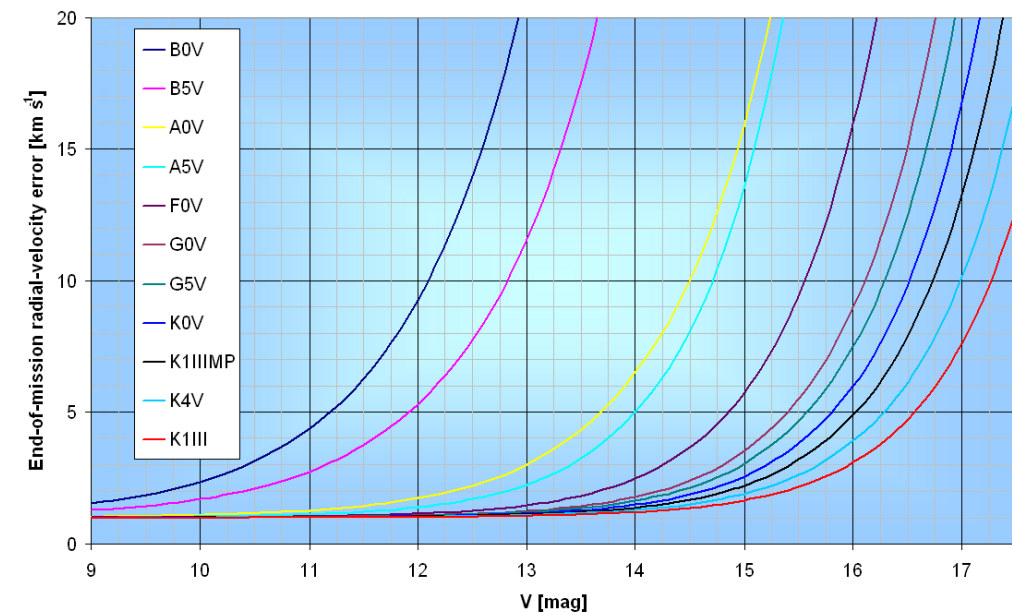
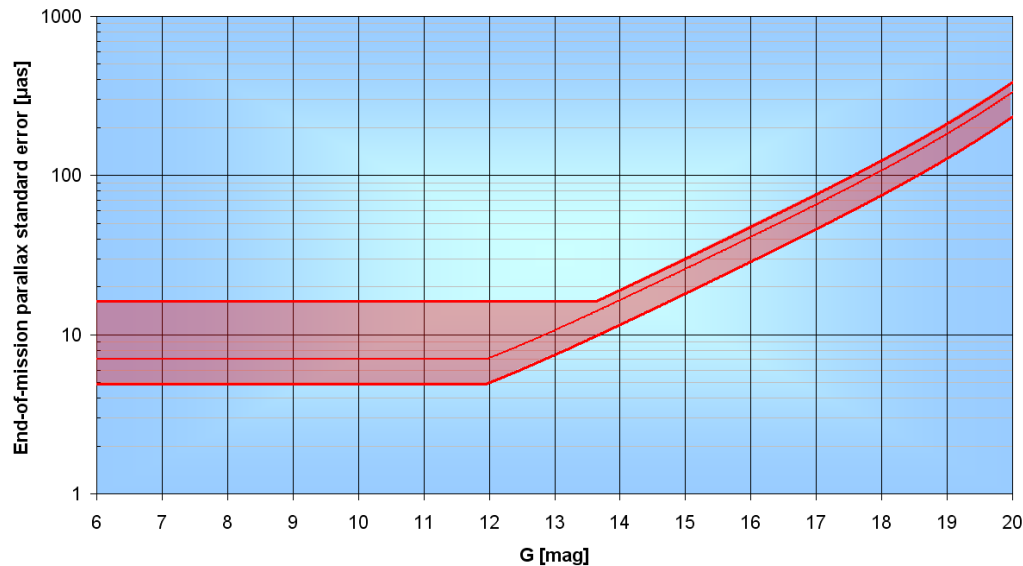
# Mock line-of-sight velocity errors



The LOSV errors can **underestimate** the LDMD when  $\beta=2-3$ .

- Gaia simulation

# Gaia simulation



Tracer stars used frequently are K dwarf stars.

– K stars

- $M \cong 7 \Rightarrow m \cong 17 @ 1 \text{ kpc}$

– parallax error = 0.06 mas

– PM error = 0.06 mas/yr

– LOSV error = 10 – 17 km/s

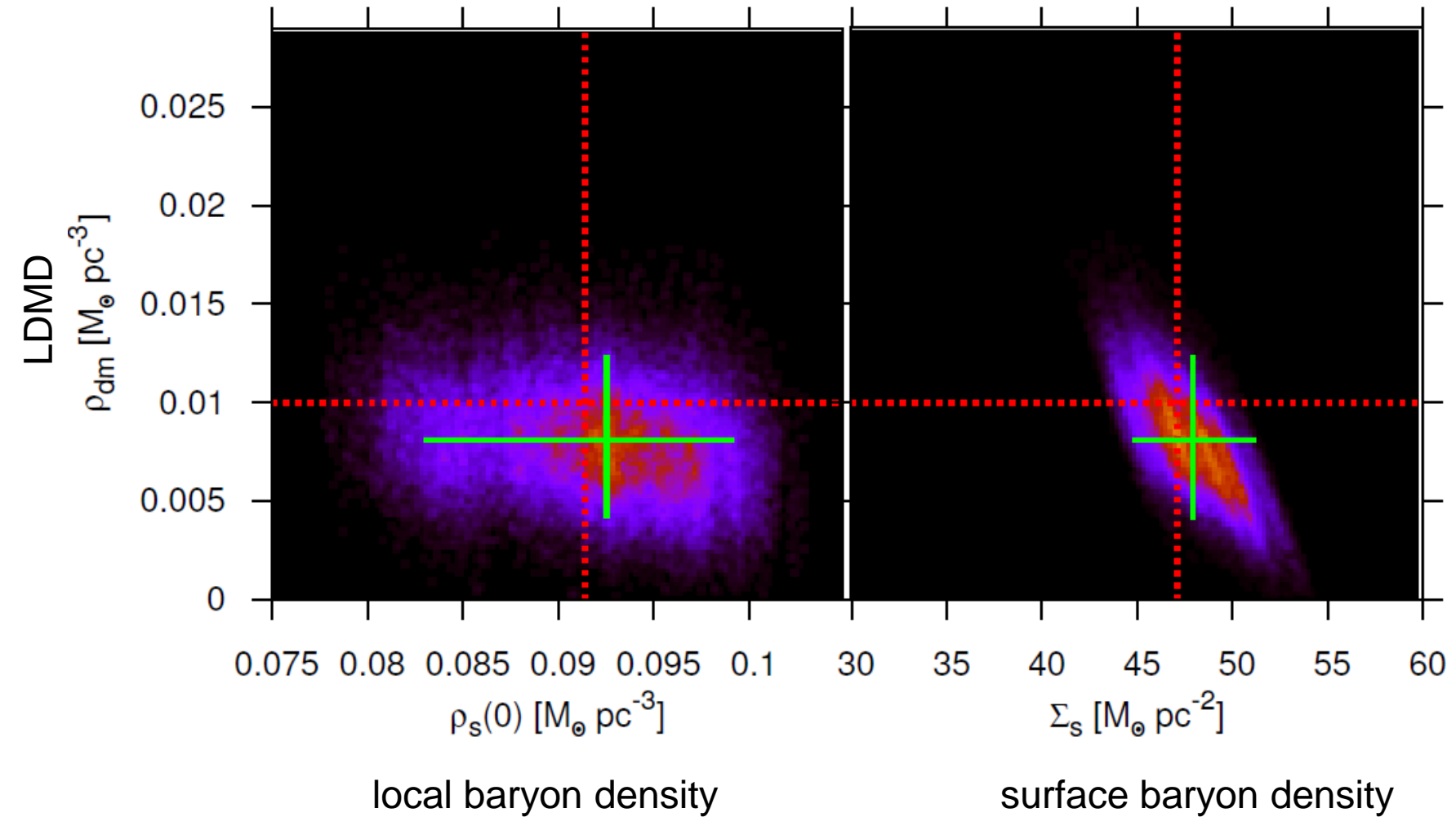
- $\beta=2-3$

–  $z_{min} = 200 \text{ pc}, z_{max} = 1.2 \text{ kpc}$

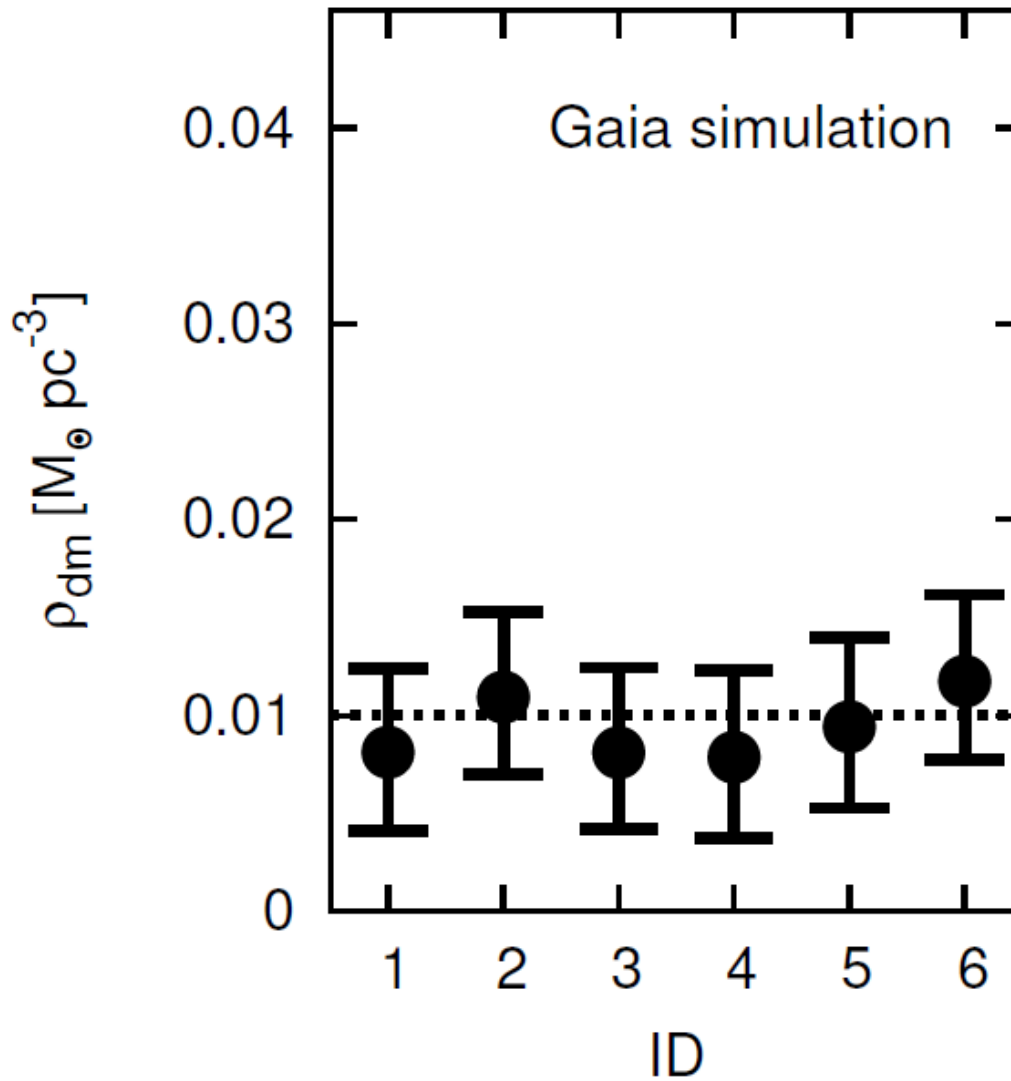
– conical region  $480 \text{ deg}^2$

– sample size = 24,000 stars

# Gaia simulation



# Gaia simulation





# Summary

- **method**
  - The MA method seems promising.
    - little or no systematic errors
      - if sample size and observational precisions are sufficient.
- **required observational precisions**
  - Parallax precision must be  $< 0.1-0.3$  mas @  $d=1$  kpc (would be a necessary condition)
    - Otherwise, distance errors can cause overestimation.
- ***Gaia & Hipparcos***
  - Gaia can exceed the required precisions.
    - Hipparcos catalogue is not sufficient.
  - **more calculations → Inoue & Gouda (2013) A&A, 555, A105**