Non-Parametric Mass Modelling of Dwarf Galaxies

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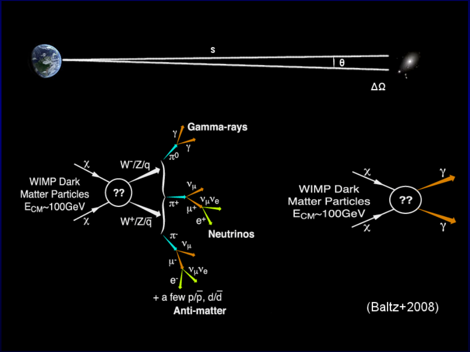
Direct DM detection



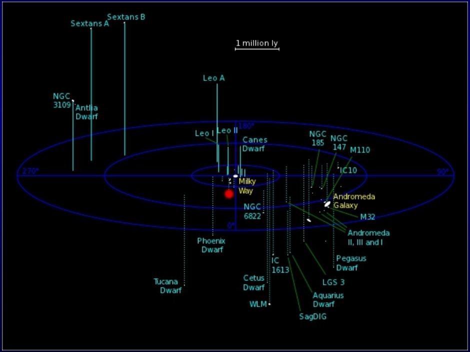
Reaction Rate

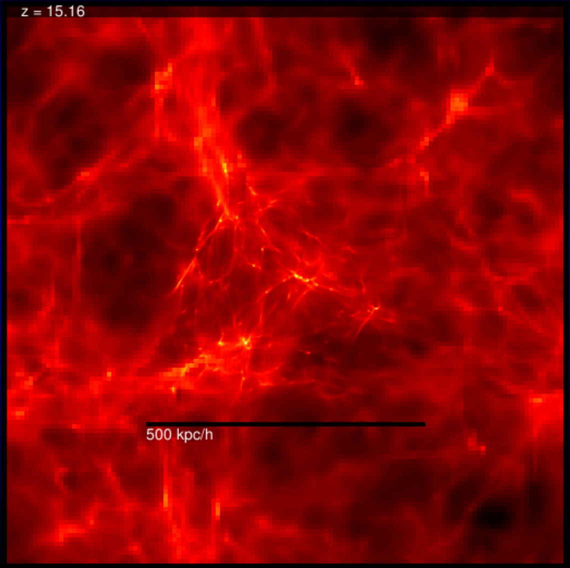
$$\frac{\mathrm{d}\Phi(\Delta\Omega, E_\gamma)}{\mathrm{d}E_\gamma} = \frac{1}{8\pi}\frac{\langle\sigma v\rangle}{m_{DM}^2}\frac{\mathrm{d}N_\gamma}{\mathrm{d}E_\gamma} \times J(\Delta\Omega)\Delta\Omega$$

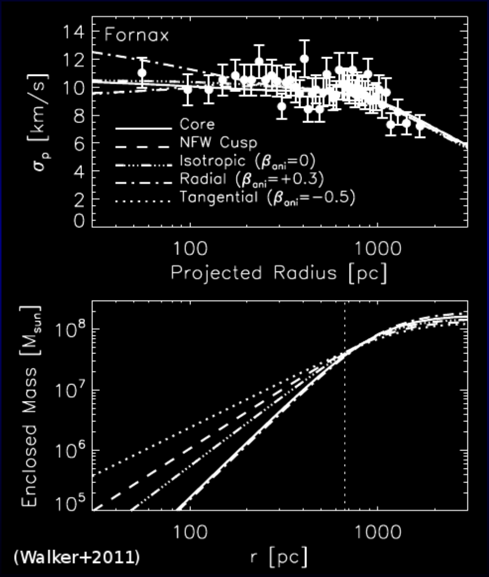
$$J(\Delta\Omega) = \frac{1}{\Delta\Omega}\int_{\Delta\Omega}\mathrm{d}\Omega\int_{\mathrm{LOS}}\mathrm{d}s\,\rho^2(r(s))$$

(Bertone+2005)

Dwarf Galaxies




z = 15.16
500 kpc/h



$$\rho_{\mathrm{NFW}} = \frac{\rho_0}{(r/r_s)(1+r/r_s)^2},$$

$$\rho_{\mathrm{ISO}} = \frac{\rho_0}{1+(r/r_c)^2}.$$

Overview Mass modelling

- Jeans modelling: 1) derive equations between ν, ρ, σ, Φ from collisionless Boltzmann equation 2) solve for Φ, and ultimately ρ_{DM} 3) restriction: ν, σ from observations
- Schwarzschild modelling: 1) postulate Φ 2) compute N orbits over M oscillations 3) reproduce ρ with population of these orbits
- DF models: Assume functional form of DF.
- Made 2 Measure: find N-body model via merit function

Jeans equations

$$0 = \frac{\mathrm{d}f}{\mathrm{d}t} = \frac{\partial f}{\partial t} + \nabla_{\vec{x}} f \cdot \vec{v} - \nabla_{\vec{v}} f \cdot \nabla_{\vec{x}} \Phi$$

$$0 = \frac{\partial f}{\partial t} + \dot{r}\frac{\partial f}{\partial r} + \dot{\theta}\frac{\partial f}{\partial \theta} + \dot{\phi}\frac{\partial f}{\partial \phi} + \dot{v}_r\frac{\partial f}{\partial v_r} + \dot{v}_\theta\frac{\partial f}{\partial v_\theta} + \dot{v}_\phi\frac{\partial f}{\partial v_\phi}$$

(Binney, Tremaine 2008)

moments:

$$\frac{1}{\nu}\frac{\partial}{\partial r}(\nu\sigma_r^2) + 2\frac{\sigma_r^2 - \sigma_t^2}{r} = -\frac{\partial \Phi}{\partial r} = \frac{GM(<r)}{r^2}$$

$+$ higher order moments of velocity, κ_{LOS}

Motivation and aims for non-parametric method

- not bound to $\rho(r) = \rho_{\alpha,\beta,\gamma,\ldots}(r)$, let data tell the form
- applicable to any gravitational model
- robust to noise in the data

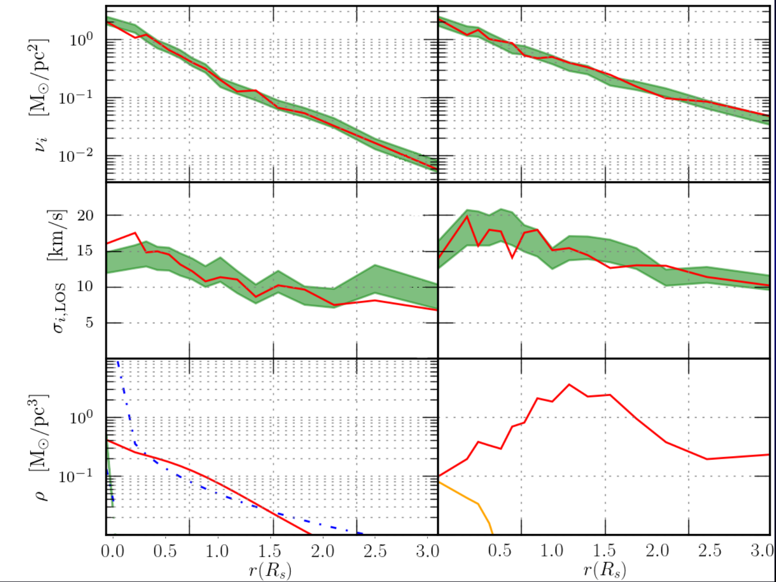
What do we mean with “non-parametric”?

- model for ρ, ν_i, $\beta_i \equiv 1 - \sigma_t^2/2\sigma_r^2$ in N bins,
- MCMC with comparison to observed 2D properties
- error function:

$$\chi^2 = \sum_i \chi^2_{\nu,i} + \chi^2_{\sigma_{\mathrm{LOS},i}} + \chi^2_{\kappa_{\mathrm{LOS},i}}$$

- fit many more model parameters (bins) than constraints, $5 \cdot N \gg n_{constraints}$
- marginalise over all bins with MCMC
- $\rightarrow$ find models that give good fits

Example Step



Algorithm Details

Priors:

- no rising density;
- total mass $\geq$ baryonic mass;
- no $\beta > 1$, "smooth" β;
- no constraints on κ

Stepsizes:

- init: set to 5% of observed values;
- adapted set-wise during burn-in phase;
- ρ parameterized as Lagrange polynomial.

M-β degeneracy

- (Merrit 1987): radial orbits: high mass; tangential orbits: low mass
- (Richardson+ 2013): higher moments $\rightarrow$ more anisotropy parameters
- (Battaglia+ 2008): two populations with different $r_{1/2}$ break degeneracy

Moments of the Jeans equations

$$
\begin{aligned}
\sigma_r^2(R) &= \frac{1}{\nu(R)} \exp\left(2\int_{r_{min}}^{r} \frac{\beta(s)}{s} ds\right) \cdot \\
&\int_R^\infty \frac{GM(<r)\nu(r)}{r^2} \exp\left(-2\int_{r_{min}}^{r} \frac{\beta(s)}{s} ds\right) dr \\
\sigma_{LOS}^2(R) &= \frac{2}{\Sigma(R)} \int_R^\infty \left(1-\beta\frac{R^2}{r^2}\right) \frac{\nu(r)\sigma_r^2(r)r}{\sqrt{r^2-R^2}} dr
\end{aligned}
$$

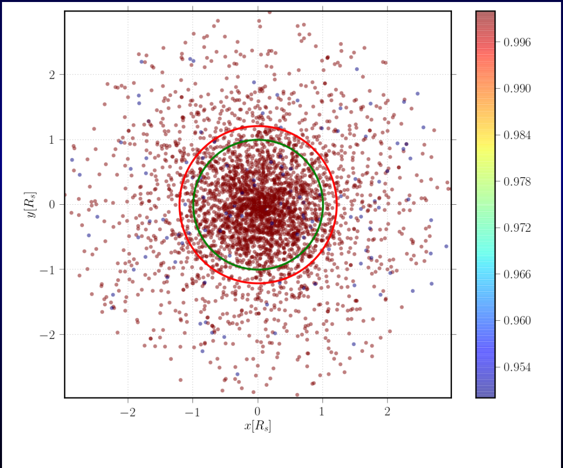
(Lokas 2005)

$+$ higher order moments

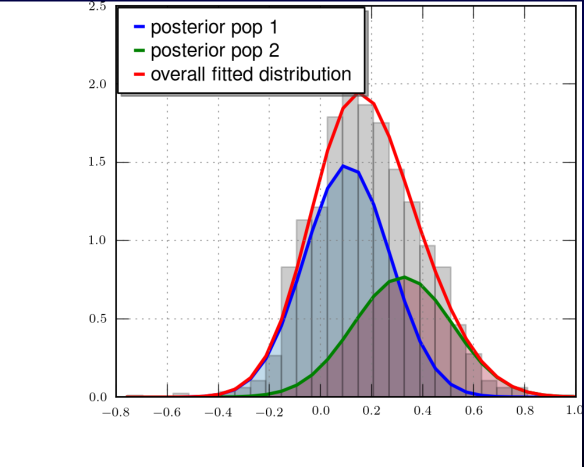
Spherical Mock Data for Gaia Challenge

- cusped / cored profile, $r_{DM} = 1000\text{pc}$
- 2 stellar tracer populations, with inner slope $\gamma_* = 1.0$
- distinguished by metallicity W_{Mg}
- different stellar scale lengths $r_* = 500, 1000\text{pc}$
- total 9000 tracers
- includes systemic shift, binary motions
- further info: (Walker 2013)

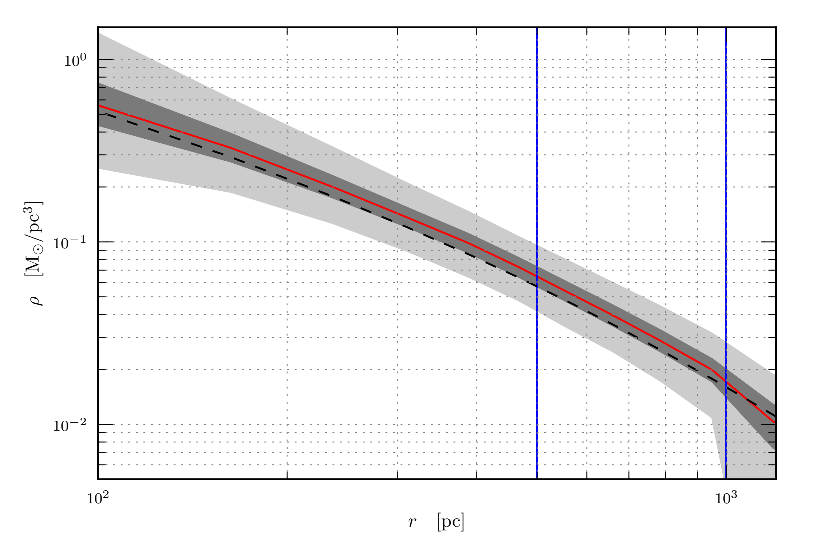
Scales, Probability of Membership



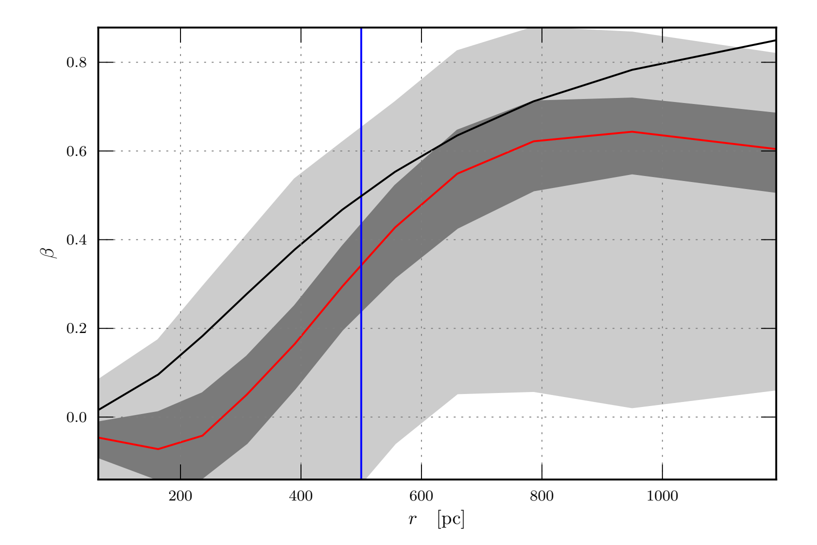
Splitting by Metallicity



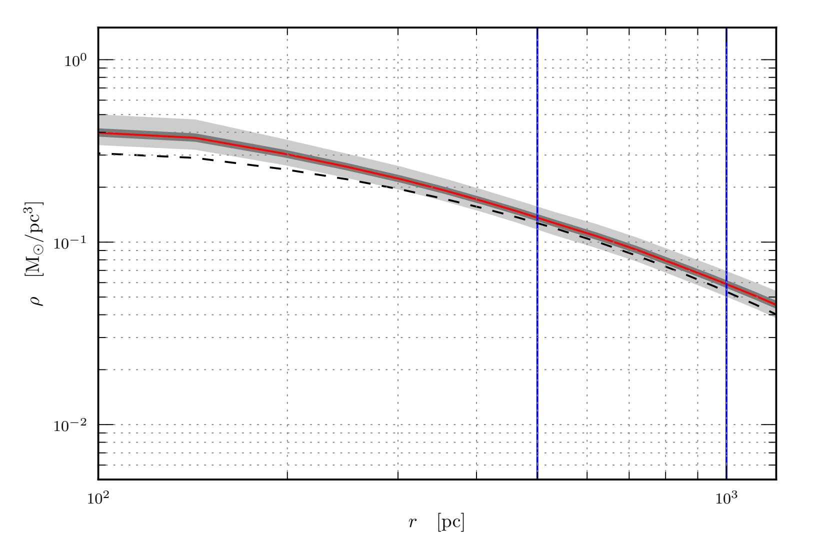
Preliminary Result: Density Profile 2 Populations



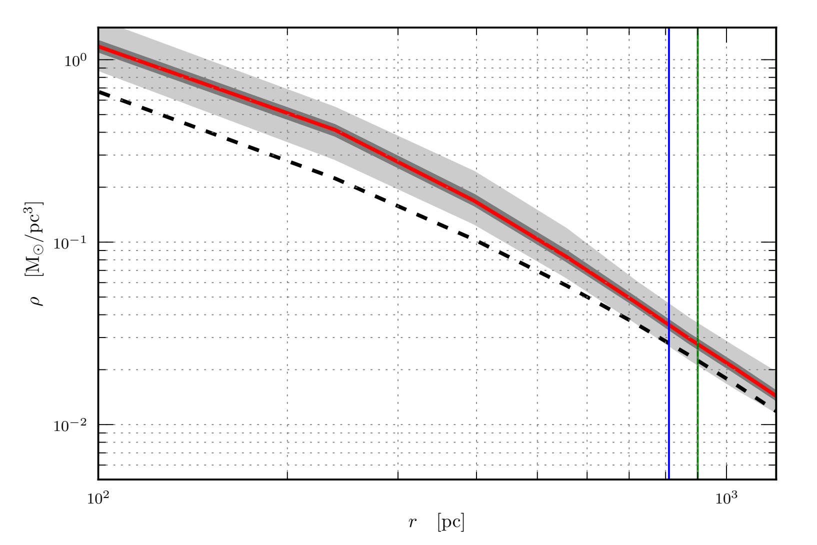
Preliminary Result: Beta Profile



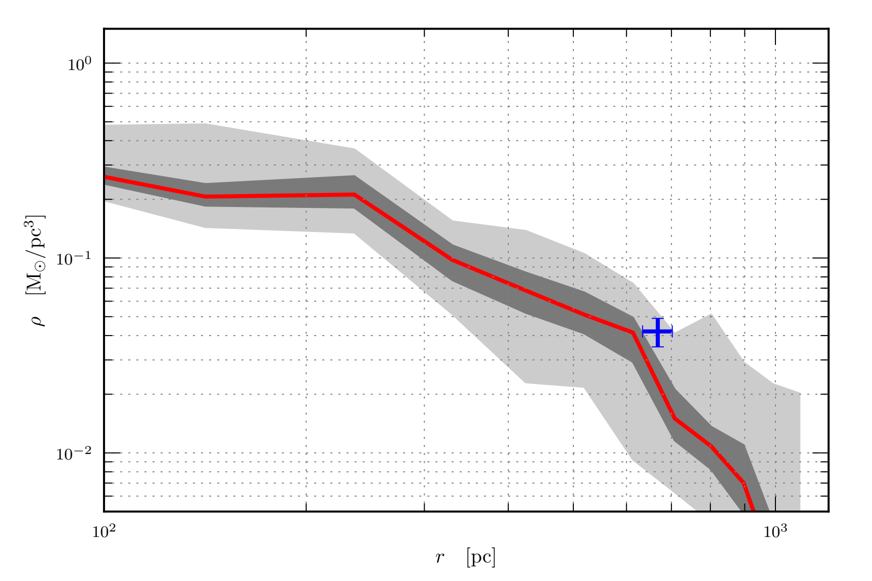
Preliminary Result: Density Profile Cored



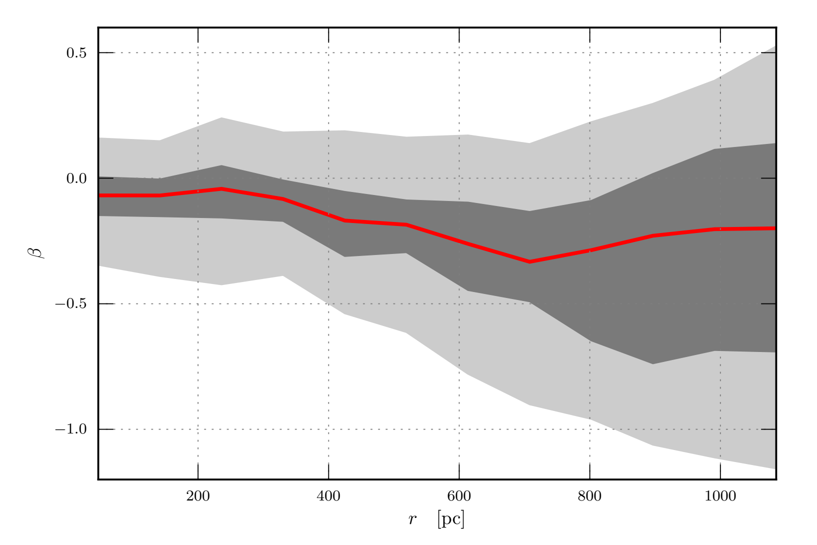
Preliminary Result: Density Profile Triaxial System



Preliminary Result: Fornax Dwarf Galaxy



Preliminary Result: Velocity Anisotropy



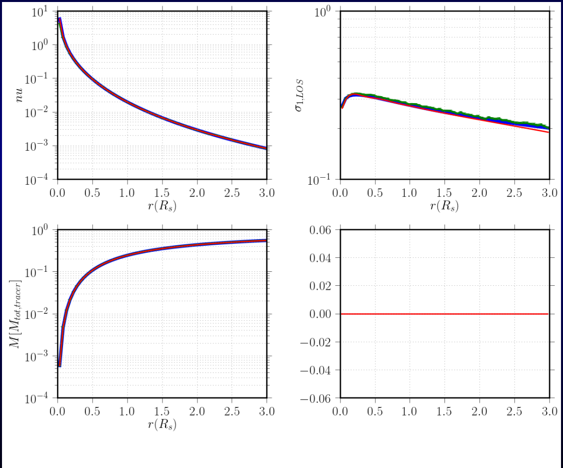
Summary

- non-parametric method: model ρ, ν, β in bins
- splitting by chemistry to break β, M-degeneracy
- dwarfs are potential places to detect dark matter annihilation signals

Outlook

- application to real observed dwarfs
- modelling of binary motions
- comparison with dwarfs in NEC simulation

Appendix: σ_{LOS} integration



Appendix: Deprojection

- work with 3D model, compare projected 2D values
- deproject, assuming spherical geometry

$$\nu = -\frac{1}{\pi}\int_R^\infty \frac{\mathrm{d}\Sigma}{\mathrm{d}R}\frac{\mathrm{d}r}{\sqrt{R^2 - r^2}}$$

Appendix: Application to Disc Geometry

- method in 1D z, working on K_z
- potential dominated by baryons near disc plane $\rightarrow$ baryon prior
- assumption: tilt term neglected